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Every Symmetric 3 x 3 Global Game of Strategic Complementarities Is Noise Independent

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EVERY SYMMETRIC 3 × 3 GLOBAL GAME OF STRATEGIC COMPLEMENTARITIES IS NOISE INDEPENDENT

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Abstract. We prove that the global game selection in all 3×3 payoff-symmetric supermodular games is independent of the noise structure. As far as we know, all other proofs of noise independence of such games rely on the existence of a so-called monotone potential (MP) maximiser. Our result is more general, since some 3×3 symmetric supermodular games do not admit an MP maximiser. Moreover, a corollary is that noise independence does not imply the existence of an MP maximiser.
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JEL codes: C72, D82.

In this note, we use methods outlined in Basteck et al. [1] to prove that the global game selection in two-player, three-action, supermodular games with symmetric payoffs is independent of the noise structure when the noise vanishes (see Frankel, Morris and Pauzner (FMP) [3] for the definition of global games used here). Games with this property are called noise independent.

Theorem. *Every* 3×3 *symmetric supermodular game is noise independent.*

Our interest in 3×3 games is piqued because they clarify the connections between the noise independence of global games, robustness to incomplete information [5], and the existence of a so-called monotone potential (MP) maximiser. As far as we know, all proofs so far of the noise independence of 3×3 symmetric supermodular games rely on the existence of an MP maximiser and only apply to the subset of games with three Nash equilibria—see Oyama and Takahashi [8] for the most general proof along these lines. Existence of an MP maximiser guarantees existence of an equilibrium robust to incomplete information [6], and *a fortiori*, noise independence—see Oury and Tercieux [7] or Basteck et al. [1]. However, Honda [4] has found a non-empty open set of symmetric 3×3 games that have no MP maximiser.

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Our proof does not rely on the existence of an MP maximiser. Since it applies to all 3×3 games with symmetric payoffs, it is necessarily more general. In particular, combined with the result of Honda, it shows that noise independence is not equivalent to the existence of an MP maximiser.[1]

Incidentally, many authors are under the impression that the noise independence of supermodular 3×3 games with symmetric payoffs was completely settled by FMP. The cases that FMP consider formally rely on the existence of an MP maximiser. But they also give a heuristic argument for the noise independence of 3×3 games with symmetric payoffs when, in addition, the noise distributions of agents' signals are symmetric in the mean. Unfortunately, it is not true[2] that if the global game selection is independent of the noise structure for all mean-symmetric noise distributions, the game is noise independent in general, as we show below per counter example.

1. Preliminaries

Consider a symmetric 3×3 game with players $i \in \{1, 2\}$, both endowed with ordered action set $A = \{a, b, c\}$, $a < b < c$ and payoff function $g : A \times A \to \mathbf{R}$, where $g(a_i, a_{-i})$ is i's payoff if she chooses a_i and her opponent a_{-i}. We may identify the game with its payoff function g. Since g is a symmetric game, we will typically denote an action profile $(a^*, a^*) \in A \times A$ also by a^*, economising slightly on notation.

Let $\Delta^{y_i}_{x_i}(x_{-i}) := g(y_i, x_{-i}) - g(x_i, x_{-i})$, the payoff difference of playing y_i instead of x_i against opposing profile x_{-i} and recall that g is called (weakly) supermodular if[3]

$$(x_i < y_i \text{ and } x_{-i} < y_{-i}) \implies \Delta^{y_i}_{x_i}(x_{-i}) \leq \Delta^{y_i}_{x_i}(y_{-i}), \tag{1}$$

in other words $\Delta^{y_i}_{x_i}(x_{-i})$ is a monotonic function for all $x_i < y_i$. A game g is called strictly supermodular if (1) still holds when the weak inequality is replaced by a strict one. The *dual* game of g, denoted g^∂, is given by reversing the ordering on the action set of g. Note that g is supermodular iff g^∂ is supermodular.

Let $f = (f_1, f_2)$ be a pair of probability densities, whose supports are subsets of $[-\frac{1}{2}, \frac{1}{2}]$. A *lower-$f$-elaboration*, $\underline{e}(g, f)$, of g, is defined as the following incomplete information game. A state parameter θ is uniformly distributed over an interval $[-\frac{1}{2}, R]$, with $R \geq 6$. Each player receives a noisy signal $x_i = \theta + \eta_i$ about the true state, with η_i drawn according to the density f_i. The random variables θ, η_1, η_2 are independently distributed. Players' payoffs u_i are given by

$$u_i(a_i, a_{-i}, x_i) := \begin{cases} \tilde{u}_i(a_i, a_{-i}) & \text{if } x_i < 0, \\ g_i(a_i, a_{-i}) & \text{if } x_i \geq 0, \end{cases}$$

with $\tilde{u}_i$ being an arbitrary payoff function that makes the least action dominant, e.g. for all a_{-i}, $\tilde{u}_i(0, a_{-i}) = 1$ and $\tilde{u}_i(a_i, a_{-i}) = 0$ when $a_i \neq 0$.

We say a strategy profile s of a lower-f-elaboration $\underline{e}(g, f)$ *attains* $a^* \in A \times A$ if $s(x) \geq a^*$ for some $x \in [-\frac{1}{2}, R]$. If s is a strategy profile of the lower-f-elaboration, and $\beta(s)$ the greatest best reply to s, we can conduct upper-best reply iterations s, $\beta(s)$, $\beta(\beta(s))$, $\beta(\beta(\beta(s)))$, … starting at some strategy profile s. If $\beta(s)$ is weakly greater than s, the resulting sequence of strategy profiles

[1]Satoru Takahashi (private correspondence) has informed us that Jun Honda's example of a symmetric 3×3 game with no MP maximiser has no equilibrium robust to incomplete information either.
[2]Nor, we should add, do FMP claim this is true.
[3]FMP use the terminology "game of strategic complementarities".

will be monotonically increasing. As the action space is bounded, this sequence converges pointwise to an equilibrium strategy profile.

An action profile $a^* \in A \times A$ is said to be *attained from below under* f if in *some* lower-f-elaboration of g, the greatest equilibrium strategy profile attains a^*. Let G be a global game with noise structure f (up to the usual scaling), such that the payoff structure equals g at some payoff parameter θ. By a theorem of Basteck et al. [1], the action profile a^* is the greatest global game selection at θ if and only if a^* is attained from below under f. An action profile is the least global game selection at θ if and and only if it is attained from below under f in g^∂, and in this case it is said to be *attained from above under* f.

2. Proof of Noise Independence of 3 × 3 Symmetric Supermodular Games

We begin by ruling out some cases. First, let us assume without loss of generality that no action dominates another (that would imply that g can be reduced to a 2×2 game known to be noise independent [2]). By supermodularity, both a and c must be equilibria. Second, we assume without loss of generality that g is strictly supermodular, as the global game selection for weakly supermodular games is pinned down by the selection in nearby strictly supermodular games.[4] Third, if b is a best reply against an opponent mixing equally over a and b as well as against an opponent mixing equally over b and c, it is the noise independent global game selection.[5]

In what follows, we analyse the remaining cases. Let us introduce the following terminology. Consider an action profile that mixes over a, b, c with probabilities ("weights") w_a, w_b, w_c. Define $S(w_c)$ to be the number w_a that solves the equation

$$(2) \quad w_a g(b,a) + (1 - w_a - w_c)g(b,b) + w_c g(b,c) = w_a g(c,a) + (1 - w_a - w_c)g(c,b) + w_c g(c,c).$$

Even though $S(w_c)$ is not necessarily in the interval $[0, 1]$, we can think of it intuitively as the weight that may be put on the least action, a, to make an agent indifferent between playing the middle action, b, and the greatest action, c, when the weight on c is w_c. Existence and uniqueness of the solution $S(w_c)$ is guaranteed by our assumptions of no dominated actions and strict supermodularity. The function S has derivative

$$\varrho_S := \frac{\Delta_b^c(c) - \Delta_b^c(b)}{\Delta_b^c(b) - \Delta_b^c(a)} > 0,$$

thus is linear and (due to supermodularity) increasing. Analogously, define $N(w_a)$ to be the weight that needs to be put on c to make the agents indifferent between playing a and b when the weight on a is w_a. That is, $N(w_a)$ is the solution for w_c to

$$(3) \quad w_a g(a,a) + (1 - w_a - w_c)g(a,b) + w_c g(a,c) = w_a g(b,a) + (1 - w_a - w_c)g(b,b) + w_c g(b,c).$$

[4] We may embed a weakly supermodular game g in a global game G where the payoff structure is symmetric and strictly supermodular for almost all θ. For example, identify $a = -1, b = 0, c = 1$, and consider a global game G where payoffs depend on a state variable θ as follows:

$$u_i(a_i, a_{-i}, \theta) := g(a_i, a_{-i}) + \theta a_i(2 + \mathrm{sgn}(\theta)a_{-i}).$$

One may verify that g is embedded at $\theta = 0$. By results in Basteck et al. [1], the global game selection in g does not depend on the embedding chosen. Since the greatest (least) global game selection is continuous in θ from the right (left), and all strictly supermodular games are noise-independent by our proof below, so is g.

[5] Such a game is "decomposable" in the sense of Basteck et al. [1]. Moreover, since b is a Nash equilibrium, it has three equilibria, thus belongs to the class that Oyama and Takahashi [8] consider.

The function N has derivative

$$\varrho_N := \frac{\Delta_a^b(b) - \Delta_a^b(a)}{\Delta_a^b(c) - \Delta_a^b(b)} > 0.$$

Lemma. *If $N(\frac{1}{2}) \le S(\frac{1}{2})$, then c is the global game selection in g.*

Proof. We will show that there exists an increasing strategy profile attaining c in a lower-f-elaboration from which an upper-best reply iteration leads upwards. In this case, c is the global game selection, by the aforementioned theorem of Basteck et al. [1]. It is easy to check that the statement is true whenever c is a best reply against an opponent mixing equally over a and c. Moreover, a cannot be a best reply to this mixture, as this would imply $N(\frac{1}{2}) > \frac{1}{2} > S(\frac{1}{2})$. Thus, we may assume without loss of generality that b is a best reply against an opponent mixing over a and c with equal probability.

Consider the following set of increasing strategy profiles in a lower-f-elaboration,

$$M := \{(\underline{z}_1, \underline{z}_2, \overline{z}_1, \overline{z}_2) \in [0,5]^4 \mid \underline{z}_i \le \overline{z}_i, \overline{z}_1 - \underline{z}_1 \le 2\},$$

where $\underline{z}_i$ denotes the threshold where player i switches from a to b, and $\overline{z}_i$ is defined analogously. If c is attained by any equilibrium strategy profile, it is attained by an equilibrium profile $s \in M$ as well.

We restrict our attention to thresholds in $[0,5]$ because the distribution over signal differences $x_i - x_{-i}$ conditional on the x_i received is the same for all $x_i \in [0,5]$. Let H be the cumulative distribution function of said signal difference $x_1 - x_2$ and without loss of generality, assume $H(0) = \frac{1}{2}$. We begin by deducing the following weights from H, which may be verified straightforwardly. Fixing $\overline{z}_1$, at player 2's threshold $\overline{z}_2$, player 2 assigns weight $w_c(\overline{z}_2) := 1 - H(\overline{z}_1 - \overline{z}_2)$ to player 1 playing c. Player 1 assigns weight $w_c(\overline{z}_1) := 1 - w_c^2(\overline{z}_2) = H(\overline{z}_1 - \overline{z}_2)$ to player 2 playing c at $\overline{z}_1$. Clearly, $w_c(\overline{z}_1)$ is continuous and increasing in the difference $\overline{z}_1 - \overline{z}_2$ and $w_c(\overline{z}_2)$ is continuous and decreasing. Moreover, $w_c(\overline{z}_1) = w_c(\overline{z}_2) = \frac{1}{2}$ when $\overline{z}_1 = \overline{z}_2$. In a similar vein, at $\underline{z}_2$, Player 2 assigns weight $w_a(\underline{z}_2) := H(\underline{z}_1 - \underline{z}_2)$ to player 1 playing a. At $\underline{z}_1$, player 1 assigns weight $w_a(\underline{z}_1) := 1 - H(\underline{z}_1 - \underline{z}_2)$ to player 2 playing a. Also, $w_c(\underline{z}_1) = w_c(\underline{z}_2) = \frac{1}{2}$ when $\underline{z}_1 = \overline{z}_2$.

For the moment, let us assume $\varrho_S \le \varrho_N$. We will define a function $F : x \mapsto y$ on the domain $[0,2]$ as follows. First, set $\underline{z}_1 = 2$ and $\overline{z}_1 = 2 + x$. Second, choose $\overline{z}_2$ equal to the least value where c becomes a best reply for player 2 against the opposing action distribution given by $\underline{z}_1 = 2$ and $\overline{z}_1 = 2 + x$. Since b is a best reply when faced with an opponent mixing over a and c with equal probability, we then have $\underline{z}_1 \le \overline{z}_2$. Also, since c is a best reply to itself, we have $\overline{z}_2 \le \overline{z}_1 + 1$, so our strategy profile will be an element of M.

Next, choose $\underline{z}_2 \le \overline{z}_2$ as large as possible under the additional constraint

$$(w_c(\overline{z}_1) - w_c(\overline{z}_2))\varrho_S \le (w_a(\underline{z}_2) - w_a(\underline{z}_1))\varrho_N. \tag{4}$$

Note that inequality (4) can always be satisfied for some $\underline{z}_2 \ge 0$: if $\overline{z}_2 > \overline{z}_1$, then $w_c(\overline{z}_1) - w_c(\overline{z}_2) \le 0$, and we may set $\underline{z}_2 = \underline{z}_1$; if $\overline{z}_2 < \overline{z}_1$, we can choose $\underline{z}_2$ such that

$$\underbrace{\overline{z}_1 - \overline{z}_2}_{\le 2} = \underbrace{\underline{z}_1}_{=2} - \underline{z}_2$$

which implies that (4) holds:

$$\begin{aligned}(w_c(\overline{z}_1) - w_c(\overline{z}_2))\varrho_S &= (2H(\overline{z}_1 - \overline{z}_2) - 1)\varrho_S \\ &\le (2H(\underline{z}_1 - \underline{z}_2) - 1)\varrho_N = (w_a(\underline{z}_2) - w_a(\underline{z}_1))\varrho_N.\end{aligned}$$

As we choose $\underline{z}_2$ as large as possible, one of the two constraints becomes binding. In addition, notice that since we choose $\underline{z}_2$ such that inequality (4) is satisfied, we must have $\underline{z}_2 \leq \underline{z}_1 + 1$. After all, when $\underline{z}_2 = \underline{z}_1 + 1$ we have $(w_a(\underline{z}_2) - w_a(\underline{z}_1)) = -1$, and our assumption that $\varrho_S \leq \varrho_N$ then entails that the reverse of inequality (4) holds.

Finally, choose $\bar{z}_1^*$ minimally, such that $\bar{z}_1^* \geq \underline{z}_1$ and c is a best reply of player 1 for signal $x_1 = \bar{z}_1^*$, given $\underline{z}_2$ and $\bar{z}_2$. We now specify $F(x)$ by putting it to $y = \bar{z}_1^* - \underline{z}_1$.

We are interested in fixpoints of F. It is easy to verify that F is continuous—as continuous changes in x change the indifference conditions used in the construction continuously. Now, consider $F(0)$, that is, the construction starting from $\underline{z}_1 = \bar{z}_1$. Since b is a best reply to an opponent mixing over a and c with equal probability, we know that $\bar{z}_2 > \underline{z}_1 = \bar{z}_1$. This implies that $w_c(\bar{z}_1) - w_c(\bar{z}_2) < 0$. If (4) is binding, we must have $w_a(\underline{z}_2) - w_a(\underline{z}_1) < 0$, so $\underline{z}_1 < \underline{z}_2$. If on the other hand $\underline{z}_2 = \bar{z}_2$, we also know know that $\underline{z}_1 < \bar{z}_2 = \underline{z}_2$. Faced with this configuration, player 1 sees an action distribution that is dominated by the distribution which mixes over a and c with equal probability. Therefore, player 1's best reply is weakly smaller than b at her threshold $\underline{z}_1$, and our construction implies $\bar{z}_1^* > \bar{z}_1$. Thus $F(0) > 0$. Next, consider $F(2)$. Since inequality (4) is satisfied, we know $\underline{z}_2 \leq \underline{z}_1 + 1$. This means that at $\bar{z}_1$, player 1 puts zero weight one her opponent playing a. Also, since the best reply to the distribution which mixes over b and c with equal probability is c, we know $w_c(\bar{z}_2) \leq \frac{1}{2}$. Hence $w_c(\bar{z}_1) = 1 - w_c(\bar{z}_2) \geq \frac{1}{2}$, and $\bar{z}_1^* \leq \bar{z}_1$.

Thus, $F(x) - x \geq 0$ when $x = 0$, and $F(x) - x \leq 0$ when $x = 2$, and from the intermediate value theorem we conclude that F has a fixpoint.

Now, let us consider a fixpoint of F and the associated strategy profile. From its construction we know that each agent prefers action c upon receiving $x_i = \bar{z}_i$. It remains to show that agents are willing to switch to b at $\underline{z}_i$. If inequality (4) is binding, then in the fixpoint the associated weights satisfy by construction:

$$\begin{aligned}(5)\quad S(w_c(\bar{z}_1)) = S(\frac{1}{2}) + (w_c(\bar{z}_1) - \frac{1}{2})\varrho_S &= S(\frac{1}{2}) + \frac{1}{2}(w_c(\bar{z}_1) - w_c(\bar{z}_2))\varrho_S \\ &\geq N(\frac{1}{2}) + \frac{1}{2}(w_a(\underline{z}_2) - w_a(\underline{z}_1))\varrho_N = N(w_a(\underline{z}_2)).\end{aligned}$$

The inequality follows since $N(\frac{1}{2}) \leq S(\frac{1}{2})$, and since $(w_c(\bar{z}_1) - w_c(\bar{z}_2))\varrho_S = (w_a(\underline{z}_2) - w_a(\underline{z}_1))\varrho_N$. Similarly:

$$(6)\quad S(w_c(\bar{z}_2)) = S(\frac{1}{2}) - \frac{1}{2}(w_c(\bar{z}_1) - w_c(\bar{z}_2))\varrho_S \geq N(\frac{1}{2}) - \frac{1}{2}(w_a(\underline{z}_2) - w_a(\underline{z}_1))\varrho_N = N(w_a(\underline{z}_1)).$$

Player 1 is indifferent at the threshold $\bar{z}_1$ if she expects a to be played with weight $S(w_c(\bar{z}_1))$. Since for a fixpoint $\bar{z}_1$ is, in fact, chosen so that player 1 is indifferent, we know that she must put weight $S(w_c(\bar{z}_1))$ on a. But the weight player 1 puts on a at her threshold $\bar{z}_1$ is $1 - H(\bar{z}_1 - \underline{z}_2)$, which is equal to the weight that player 2 puts on c at her threshold $\underline{z}_2$. The first inequality now says that this is sufficient weight to make b a best reply for player 2 at $\underline{z}_1$. From the second inequality, we similarly deduce that player 1's best reply at $\underline{z}_1$ is b. Thus, by construction, the thresholds constitute a strategy profile from which an upper-best reply iteration will lead upwards.

If condition (4) does not hold with equality, then we know that $\underline{z}_2 = \bar{z}_2$ holds instead, so that at $x_2 = \underline{z}_2 = \bar{z}_2$ player 2 is indifferent between b and c and prefers both over a. The reasoning for why player 1 prefers b at $\underline{z}_1$ is analogous to the reasoning above, using (6) and the fact that (4) holds with inequality.

Now, if $\varrho_S \geq \varrho_N$ we can apply an analogous argument. We define $F : x \mapsto y$ as follows. First, set $\bar{z}_1 = 3$ and $\underline{z}_1 = 3 - x$. Second, choose $\underline{z}_2$ equal to the greatest value where a becomes a best reply for player 2 against the opposing action distribution given by $\underline{z}_1 = 3 - x$ and $\bar{z}_1 = 3$. Since b is a best reply when faced with an opponent mixing over a and c with equal probability, we must have $\underline{z}_2 \leq \bar{z}_1$.

Next, choose $\bar{z}_2$ such that $\underline{z}_2 \leq \bar{z}_2$ and as small as possible under the additional constraint

$$(w_c(\bar{z}_1) - w_c(\bar{z}_2))\varrho_S \geq (w_a(\underline{z}_2) - w_a(\underline{z}_1))\varrho_N. \tag{7}$$

Again inequality (7) can always be satisfied, and as we choose $\underline{z}_2$ as small as possible, one of the two constraints must be binding.

To complete the specification of F, choose a new value $\underline{z}_1^*$ such that $\underline{z}_1^* \geq \bar{z}_1$ equals the greatest value where a becomes a best reply of player 1, given $\underline{z}_2$ and $\bar{z}_2$, and put $y = \bar{z}_1^* - \underline{z}_1$.

Again, one may verify F has a fixpoint. Consider a fixpoint of F. If inequality (7) is binding, then inequalities (5) and (6) hold by construction. Player 2 is indifferent at the threshold $\underline{z}_2$ if she expects c to be played with weight $N(w_a(\underline{z}_2))$. Since in a fixpoint $\underline{z}_2$ is, in fact, chosen so that player 2 is indifferent, we know that she must put weight $N(w_a(\underline{z}_2))$ on c. But the weight player 2 puts on c at her threshold $\underline{z}_2$ is exactly equal to the weight that player 1 puts on a at her threshold $\bar{z}_1$. The first inequality now says that this is less weight than is needed to make b a best reply for player 1 at $\bar{z}_1$, thus player 1's best reply at $\overline{z_1}$ is c. From the second inequality, we may similarly deduce that player 2's best reply at $\bar{z}_2$ is c. Therefore, by construction, the thresholds constitute a strategy profile from which an upper-best reply iteration will lead upwards.

If (7) doesn't hold with equality, then we still know from our construction that player 2 is indifferent at $\underline{z}_2 = \bar{z}_2$ between a and b and prefers both over c. The reasoning for why player 1 prefers b at $\underline{z}_1$ is analogous to the reasoning above, using (5) and the fact that (7) holds with inequality. □

Corollary. *If $N(\frac{1}{2}) \geq S(\frac{1}{2})$, then a is the global game selection in g.*

Proof. In the dual game of g, the ordering on A is reversed. Define N^∂ and S^∂ for g^∂ analogous to N and S for g, by replacing all the occurrences of a in expressions (2) and (3) by c, and all occurrences of c by a. We find that $N^\partial = S$ and similarly $S^\partial = N$, and therefore $N^\partial(\frac{1}{2}) \leq S^\partial(\frac{1}{2})$. By our lemma, a is the noise independent selection in g^∂. Since g and g^∂ differ only in their ordering, a is the noise independent selection in g as well. □

Together, the lemma and its corollary complete our analysis of the remaining cases, proving the theorem.

Remark. It may be verified that the payoff conditions given by FMP lead to the same prediction of the global game selection, even when applied to games they do not formally consider (such as games with two equilibria).

3. Mean-symmetric noise independence versus noise independence

We now consider whether noise independence against mean-symmetric noise distributions implies noise independence. As symmetric supermodular 3×3 games are noise independent, we turn to the asymmetric 3×3 game in figure 1. Both agents are indifferent between a and b when facing an opponent who plays (a, b, c) with probabilities $(\frac{1}{2}, \frac{1}{6}, \frac{1}{3})$ and indifferent between b and c when facing a probability distribution $(\frac{1}{3}, \frac{1}{6}, \frac{1}{2})$. We will see, that in any lower-f-elaboration for a symmetric noise distribution f, we can find threshold values $\underline{z}_1, \bar{z}_1, \underline{z}_2, \bar{z}_2$ where agents switch to b and c such that they hold the above mentioned beliefs over opponents play and are indifferent at each switchpoint. Thus, a is attained from above under f, and c is attained from below under f, and the example is a knife-edge case where both a and c are part of the global game solution.

This is generally no longer possible if the noise distribution is asymmetric and we will present an example where agents can be made indifferent only at three switchpoints, while one agent is not willing to switch at the last remaining threshold. Thus, c is not part of the global game solution.

By perturbing the payoff table slightly, we could create a game where c is the unique global game solution for symmetric noise, while the asymmetric noise example would still uniquely select a, but in order to keep things simple, we will stick to the numbers above.

Symmetric noise. Without loss of generality let us assume that the conditional densities over the opponents signal are symmetric at 0. Set $\underline{z}_1, \underline{z}_2 = 0$. Then both agents expect their opponent to play a with probability $\frac{1}{2}$. Next, set $\bar{z}_{-i}$ such that on receiving a signal $x_i = 0$ an agent expects b to be played with probability $\frac{1}{6}$ and c with probability $\frac{1}{3}$. Due to symmetry, we find that $\bar{z}_1 = \bar{z}_2 = t$ so that an agent at $x_i = t$ holds belief $(\frac{1}{3}, \frac{1}{6}, \frac{1}{2})$ over (a, b, c) being played.

Asymmetric noise. Without loss of generality assume that agents assign probability $\frac{1}{2}$ to the event that their opponent receives a signal smaller that their own. Set $\underline{z}_1 = 0$. Adjust $\underline{z}_2$ such that agent 2 is indifferent between a and b: this is the case for $\underline{z}_2 = \underline{z}_1 = 0$, irrespective of $\bar{z}_1, \bar{z}_2$. Next, adjust $\bar{z}_1, \bar{z}_2$ simultaneously to a level where agents are indifferent between b and c. In general, we will find that $\bar{z}_1 \neq \bar{z}_2$, so the probability that agent 2 assigns to her opponent playing c will be unequal to $\frac{1}{2}$. But this implies that the probability she assigns to agent 1 playing a will be unequal $\frac{1}{3}$. For agent 1 with signal $\underline{z}_1 = 0$ this implies that while she assigns probability $\frac{1}{2}$ to agent 2 playing a, she assigns a probability unequal $\frac{1}{3}$ to agent 2 playing c. Thus, she strictly prefers either a or b so that we are no longer in knife-edge territory and the global game solution is either a or c, uniquely.

For a numerical example consider the following conditional density of player 1 over player 2's signal:

$$\pi_1(x_2|x_1) := \begin{cases} 1 + x_2 - x_1 & \text{if } x_1 - 1 < x_2 < x_1, \\ x_2 - x_1 & \text{if } x_1 < x_2 < x_1 + 1 \end{cases}.$$

Agent 2 holds a mirrored version, namely

$$\pi_2(x_1|x_2) := \begin{cases} x_2 - x_1 & \text{if } x_2 - 1 < x_1 < x_2, \\ 1 + x_2 - x_1 & \text{if } x_2 < x_1 < x_2 + 1 \end{cases}.$$

By numerical methods we establish that $\bar{z}_1 \simeq 0.22138$, $\bar{z}_2 \simeq 0.522415$. Thus probability of c at $\underline{z}_1$ is approximately equal to $0.5 - 0.5(0.522415)^2 = 0.3635 > \frac{1}{3}$. In this case, a is uniquely selected. The π_i's may be hard to generate using FMP's global game information structure. However,

		player 2: a	b	c
player 1	a	2, 1	0, 0	−3, −3
	b	0, −1	0, 0	0, 0
	c	−3, −1	0, 0	2, 2

FIGURE 1. Asymmetric two-player three-action game

they can be approximated close enough for the numerical result to hold: assume that agent 1 receives an arbitrarily precise signal, while agent 2's signal is distributed around θ just like x_2 is distributed around x_1 according to π_1.

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